**Shayabantu High School**

**GRADE 9 MATHEMATICS**

**TERM 2**

**FORMAL ASSESSMENT TASK INVESTIGATION:**

**FACTORIZATION**

Memorandum

**Total: 50 Marks Time: 1½ hour**

**Instructions:**

1. Write your name and surname in the spaces above.
2. All questions must be answered on the question paper.
3. Show all calculations.
4. You may use an approved calculator.
5. You must do your own work.
6. Check your answers.
7. Show the units of measurement where applicable.
8. All answers must be rounded to one decimal place unless stated otherwise.

**ACTIVITY 1**

'Factor' is a term used to express a number as a product of any two [numbers](https://www.cuemath.com/numbers/).

Factorization is a method of finding factors for any mathematical object, be it a number, a [polynomial](https://www.cuemath.com/algebra/polynomials/)or any algebraic expression.

Thus, factorization of an algebraic expression refers to finding out the factors of the given algebraic expression.

Example:

**The factors of 10 are 1,2,5, and 10.**

An algebraic expression can also be factorized. When the factors are multiplied they result in the original number or an expression that is factorized.

Example:

**The factors of** 7mn2 **are** m, n, n2, mn, mn2, 1, 7, 7m, 7n, 7n2, 7mn and 7mn2.

1. The following algebraic expression is given:



* 1. Write down all the possible factors for each of the following expressions.

$3a^{2}b$=$3, a$,$a^{2}$ ,$3a^{2}, 3a$ , $ab, b,a^{2}b , 3a^{2}b , 3ab √$(1)

$4ab^{2}$= $4$, $4a, 4ab, b$, $ab^{2}$, $b^{2}, a , 4b^{2}, ab$$√$(1)

* 1. Determine the H.C.F. of the algebraic expression.

 $ab$ $√$ (1)

Equivalent expressions

Two expressions are said to be **equivalent**if they have the same value irrespective of the value of the variable(s) in them.

The algebraic expressions shown below can be described as equivalent algebraic expressions.

1. $8m-16=8 ( m-2 )$
2. $6a+12= $6( a + 2 )
3. $9ab+18a=9a ( b+2 )$
	1. Write the expression **3a2b + 4ab2**as an equivalent algebraic expression.

 $ab√ \left(3a+4b\right)√ $(2)

 1.3 Lucy is given the expression below:

$$7x+35$$

Explain to Lucy how to find the equivalent expression.

Find the common factor in both terms:

 $$(7×x)+(7 ×5 ) √$$

The common factor is 7 .

|  |
| --- |
|  |

By the distributive property: ab + ac = a(b +c) Hence, an equivalent expression for the given is when you factor out the common factor of the two terms.

$$7( x+5 ) √$$

(2)

**[Total:7]**

**ACTIVITY 2**

The following algebraic expression is given:

$$xy + 5y + 2x + 10$$

* 1. What do you notice about all 4 terms?

There is no common factor in all 4 terms $√$(1)

* 1. If we group the first two terms and the last two terms as follows:

($xy  + 5y)  + (2x + 10)$

 Group 1 Group 2

what do you notice about each group?

Each group has a common factor, in group 1 the common factor is y and in group 2 the common factor is 2 $√$ (1)

* 1. Factor these values out of each group and then write down the equivalent algebraic expression.

$y(x  + 5)  + 2(x + 5)$ $√$ (1)

* 1. What is the common factor in the two terms?

 $(x  + 5) √ $ (1)

* 1. Use the distributive property to factor out this common factor and then express the polynomial as a product of two binomials.

$(x  + 5)  ( y+2 )√$(1)

* 1. Joseph wants to factorize the following algebraic expression:

$$3x^{2}  + 6x + 4x + 8$$

Provide Joseph with a three- step guide on how to factorize the expression.

 Step 1: Group the first two terms together and then the last two terms together.

$$(3x^{2}  + 6x) +(4x + 8) √$$

Step 2: Factor out a common factor from each separate binomial.

$$3x(x  + 2) +2(x + 2) √$$

Step 3: Factor out the common binomial

$$(x  + 2) +(3x + 2) √$$

 (3)

**[Total:8]**

**ACTIVITY 3**

Factorizing Trinomials in the form: $x ^{2}+ bx + c$

$$x ^{2}+ bx + c$$

Find two integers, $r $and $s$, whose **product** is $c$and whose **sum** is $b$ to rewrite the trinomial as:

$$ x ^{2}+ rx + sx + c$$

3.1 Factorizing $ x^{2} + 5x + 6$

 3.1.1 What is the value of b and c in the trinomial?

b=5 (1)

c =6 (1)

 3.1.2 Use the table below to determine the two integers, $r $and $s$.

|  |  |  |  |
| --- | --- | --- | --- |
| Factors of **6** | Product of the two factors  | Sum of the two factors | Result |
| $1 and 6$  | $$ 1 × 6 = 6$$ | $$1 + 6 = 7$$ | $$product = 6 but sum \ne 5$$ |
| $$-1 and -6$$ | $$-1 × -6 = 6$$ | $$-1+ -6 = -7$$ | $$product = 6 but sum \ne 5$$ |
| $$2 and 3$$ | $$2 × 3 = 6$$ | $$2+ 3 = 5$$ | $$product = 6 and sum =5$$ |
| $-2 and -3$  | $$-2 × -3 = 6$$ | $$-2+ -3 =-5$$ | $$product = 6 but sum \ne 5$$ |

 $√$ $√$ $√$ (3)

Which two integers will correctly provide the values of b and c in the expression

 $ x^{2} + 5x + 6?$

2$√$ and 3$√$ (2)

3.1.3 Rewite $ x^{2} + 5x + 6$ as an equivalent expression in the form  $ x^{2} + rx +sx+ 6 $.

$ x^{2} + 2x+3x + 6$ $√$(1)

3.1.4 Use the knowledge obtained from activity 2 on grouping and the distributive property to factorize the expression.

 $ x^{2} + 2x+3x + 6$

= $ (x^{2} + 2x)+(3x + 6)$ $√$

= $x\left(x+ 2\right)+3\left(x + 2\right)√$

= $(x+ 2)+(x + 3)$ $√$ (3)

3.2 Factorizing $x^{2} + x – 12$

3.2.1 What is the value of b and c in the trinomial?

b=1 $√$(1)

c =-12 $√$ (1)

 3.2.2 Use the table below to determine the two integers, $r $and $s$.

|  |  |  |  |
| --- | --- | --- | --- |
| Factors of -12 | Product of the two factors  | Sum of the two factors | Result |
| 1 and -12 | $$ 1 ×(-12 )= -12$$ | $$1 +\left(-12\right)= -11$$ | $$product = -12 but sum \ne 1$$ |
| -1 and 12 | $$ -1 ×12 = -12$$ | $$-1 +12= 11$$ | $$product = -12 but sum \ne 1$$ |
| 2 and -6 | $$ 2 ×(-6 )= -12$$ | $$2 +\left(-6\right)= -4$$ | $$product = -12 but sum \ne 1$$ |
| -2 and 6 | $$ -2 ×6= -12$$ | $$-2 +6= 4$$ | $$product = -12 but sum \ne 1$$ |
| 3 and -4  | $$ 3 ×(-4)= -12$$ | $$3 +\left(-4\right)= -1$$ | $$product = -12 but sum \ne 1$$ |
| -3 and 4  | $$ -3 ×(4 )= -12$$ | $$-3 +4= 1$$ | $$product = -12 and sum=1$$ |

 $√$ $√$ $√$ $√$ (4)

3.2.3 In the table below, the written explanation of the steps have been provided, show the mathematical steps for the explanations given.

|  |
| --- |
| **Factorize  *x*2 + *x* – 12** |
| **M**athematical steps | Explanation |
|  $x^{2} + 4x - 3x – 12$ | Rewrite the middle term of the trinomial using the values from the chart above.  |
| $$( x^{2}+ 4x) – (3x + 12) √$$ | Group pairs of terms. |
| $x( x + 4) – (3x + 12)√$ |  Factor out the HCF of the first group. |
|  $x( x + 4) – 3(x + 4)√$ |  Factor out the HCF of the second group. |
|  $$( x + 4) (x – 3) √$$  |  Factor out HCF of the two terms |
| (x + 4)(x – 3) | The final factorized answer  |

  (4)

3.3 Julie wants to factorize the following trinomial $x^{2}+9x+20,$ provide her with 3 easy steps to follow.

Step 1 : Comparing $x^{2}+9x+20$ with the standard form of $ax^{2}+bx+c$ , we get, b = 9,

 and c = 20$√$

Step 2: Find the paired factors of c such that their sum is equal to b. The suitable pair factor .

 is 4 and 5.$√$

$$x^{2}+5x+4x+20$$

Step 3 : Use grouping and the distributive property to factorize.

$$x^{2}+5x+4x+20$$

$$x\left(x+5\right)+4\left(x+5\right)$$

$\left(x+5\right)+\left(x+4\right)√$ (3)

Factorizing Trinomials in the form: $ax ^{2}+ bx + c$

Not all trinomials look like $x^{2} + bx + c,$ where the coefficient in front of the $x^{2}$ term is 1.

For the trinomials that look like $ax ^{2}+ bx + c$ , the first step should be to look for common factors of the three terms.

3.4.1 Complete the table below by identifying the common factor in each trinomial and then writing the trinomial by factoring out the common factor.

|  |  |  |
| --- | --- | --- |
| **Trinomial** | **What is the common factor ?** | **Write the trinomial with the Common Factor factored out.** |
| $$2x^{2} + 10x + 12$$ | 2 | $$2(x^{2} + 5x + 6)$$ |
| $$-5a^{2} - 15a - 10$$ | -5 | -5( $a^{2} +3a +2$) |
| $$c^{3} – 8c^{2}+ 15c$$ | *c* | $$c(c^{2} – 8c+ 15)$$ |

  $√$ $ √$ (2)

3.4.2 In the table below, the mathematical steps to factorize the trinomial has been provided.

Provide an explanation for each step.

|  |
| --- |
| **Factorize 3*x*3 – 3*x*2 – 90*x*.** |
| **Step** | **Mathematical steps** | **Explanation** |
|  1 | $$3(x^{3} – x^{2}– 30x)$$ | Factor out 3 which is common. |
|  2 | $$3x(x^{2} – x– 30)$$ | $x$ is also a common factor, so factor out $x$.$√$ |
|  3 |     3*x*(*x*2 – 6*x* + 5*x* – 30) | To find $r$ and $s$, identify two numbers whose product is $-30$ and whose sum is $-1$.The pair of factors is −6 and 5. So replace $–x $with $-6x + 5x. √$ |
|  4 | 3*x*[(*x*2 – 6*x*) + (5x – 30)] | Use grouping to consider the terms in pairs.$√$ |
|  5 | 3*x*[(*x*(*x* – 6) + 5(*x* – 6)] | Factor $x$ out of the first group and factor $5$ out of the second group.$√$ |
|  6 |  3*x*(*x* – 6)(*x* + 5) |  Then factor out $x – 6. √$ |
|  *7* | 3*x*(*x* – 6)(*x* + 5) |  The final answer is written as a product of factors. |

 (5)

3.5 Alice was provided with the following trinomial:

$3x^{2} + 7x-12x -34-2x^{2}+1$**0**

3.5.1 Provide Alice with a step-by-step guide on how to factorize the algebraic expression.

Step 1 : Collect like terms

$x^{2} -5x -24$$√$

Step 2 : Comparing $x^{2}-5x+24$ with the standard form of $ax^{2}+bx+c$ , we get, b = -5,

 and c = -24$√$

Step 3: Find the paired factors of c such that their sum is equal to b. The suitable pair factor

 is 3 and -8 .

$$x^{2}+3x-8x-24√$$

Step 4 : Use grouping and the distributive property to factorize

$$x^{2}+3x-8x-24$$

$$x\left(x+3\right)-8\left(x+3\right)$$

$\left(x+3\right)+\left(x-8 \right)√$ (4)

**[Total :35**]